The results of the calculations show that, with definite combinations of γ and ε, around 25% of the flow rate along the channel may circulate in the transversal direction. In real situations, such intensive circulation may have a decisive influence on the heat-transfer characteristics in the rotating channel with permeable walls.

NOTATION

x, y, z, Cartesian coordinates; u, v, w, components of the velocity of relative motion; p^*, modified pressure; h, channel halfheight; ω, angular velocity of rotation of the channel; ν, kinematic viscosity; w_m, mean velocity over the flow rate in the direction of the z axis; v_0, injection rate; $\varepsilon = v_0 h/\nu$, Reynolds number based on the injection rate; $\gamma = h(\omega/\nu)^{1/2}$, rotational parameter; $\xi = y/h$, dimensionless coordinate; $\bar{u} = u/\omega_m$, $\bar{w} = w/w_m$, dimensionless velocity components; c, ζ, complex quantities; f, ratio of the longitudinal component of friction to the corresponding quantity obtained with $\varepsilon = 0$ and the same γ; q, ratio of the flow rate circulating in the transverse transversal direction – Eq. (12) – to the corresponding quantity calculated for the same γ with $\varepsilon = 0$.

LITERATURE CITED

1. V. M. Eroshenko and L. I. Zaichik, Hydrodynamics and Heat Transfer of Porous Surfaces [in Russian], Moscow (1984).
2. A. S. Berman, J. Appl. Phys., 29, No. 1, 71-75 (1958).
3. O. N. Ovchinnikov and E. M. Smirnov, Inzh.-Fiz. Zh., 35, No. 1, 87-92 (1978).
4. V. Vidyanidhi, V. Bala Prasad, and V. V. Ramana Rao, J. Phys. Soc. Jpn., 39, No. 4, 1077-1081 (1975).

NONSTEADY NONISOTHERMAL FLOW OF COMPRESSIBLE GAS IN A CHANNEL WITH TRACK SAMPLING

B. F. Glikman and V. A. Gur'ev

UDC 536.253

A method is proposed for forming a linear mathematical model of a pulsating gas flow with entropy waves, in the case of gas sampling distributed along a channel, and the sampling are compared with experimental data.

Linearized equations of hydromechanics, heat transfer, and chemical kinetics are used for dynamic analysis, the determination of the acoustic and vibrational loads, and calculation of the stability of processes in power, transport, chemical-engineering, and other equipment [1, 2]. A method of constructing linear mathematical models of nonsteady isothermal gas flow in a cylindrical channel with distributed sampling is considered. Examples of such a channel may be a gas pipeline with take-off to the customer, a gas-distribution system in motors, the collector of a turbine nozzle apparatus with gas supply through a single tube and take-off over a ring at the nozzle, etc. Nonisothermal nonsteady gas flow in a cylindrical channel is described by the following system of linearized equations for dimensionless deviations (variations) of the parameters, neglecting viscosity, heat conduction, and diffusion and assuming that $D \ll L$ and the one-dimensional approximation may be used.

$$\frac{\partial \delta u}{\partial t} + u\frac{\partial \delta u}{\partial x} + \frac{p}{\rho u}\frac{\partial \delta p}{\partial x} = 0;$$
$$\frac{\partial \delta p}{\partial t} + u\frac{\partial \delta p}{\partial x} + \frac{\rho u c^2}{p}\frac{\partial \delta u}{\partial x} = 0; \quad (1)$$
$$\frac{\partial \delta s}{\partial t} + u\frac{\partial \delta s}{\partial x} = 0.$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 4, pp. 543-550, April, 1988. Original article submitted November 8, 1986.

In examining the dynamics and stability of the process, consideration may be restricted to the problem of induced oscillations, i.e., particular periodic solutions of Eq. (1): $\delta p = \delta\bar{p}\exp(i\omega t)$, $\delta u = \delta\bar{u}\exp(i\omega t)$, $\delta s = \delta\bar{s}(\exp(i\omega t)$; the amplitude of oscillations of the parameters $\delta\bar{p}$, $\delta\bar{u}$, $\delta\bar{s}$ are complex parameters in the general case, depending on the specified frequency ω. The particular periodic solution of Eq. (1) relating the variation in gas-flow parameters at the input and output of the cylindrical section of the channel is conveniently written in the form of hexapole equations [1, 2]. In the theory of the system [3], an n-pole is an element described by n variables related by n/2 linear equations. For a nonsteady nonisothermal gas flow in a channel, its dynamic characteristics are conveniently described in a linear approximation by means of a hexapole [1], the equations of which relate six variables: the deviations of the pressure, velocity (flow rate), and temperature (entropy) at the input and output of the channel section.

For a cylindrical section of channel, the induced oscillations in the gas flow are described by particular periodic solutions for each i-th variation $\delta x_i = \delta\bar{x}_i\exp(i\omega t)$. Such a solution of Eq. (1) is written in matrix form by the hexapole equation in A parameters [1]

$$\begin{bmatrix} \delta\bar{u}_{i0} \\ \delta\bar{p}_{i0} \\ \delta\bar{s}_{i0} \end{bmatrix} = \begin{bmatrix} F+H & (F-H)/\varkappa M & 0 \\ \varkappa M(F-H) & F+H & 0 \\ 0 & 0 & \exp(i\omega L/u) \end{bmatrix} \begin{bmatrix} \delta\bar{u}_{i1} \\ \delta\bar{p}_{i1} \\ \delta\bar{s}_{i1} \end{bmatrix} = [A_i][\delta\bar{u}_{i1}\ \ \delta\bar{p}_{i1}\ \ \delta\bar{s}_{i1}]^T, \quad (2)$$

where $F = 0.5\exp[i\omega L/(c+u)]$; $H = 0.5\exp[-i\omega L/(c-u)]$; [A] is the hexapole matrix; T is the transposition index. Equation (2) describes the propagation of acoustic and entropy waves in a subsonic one-dimensional gas flow. When using the multipole method, the gas channel is divided into a series of sections, for each of which the relation between the gas parameters at the input and output is known: a cylindrical section, fork, local resistance, etc. For the cylindrical section, Eq. (2) of the linear mathematical model is obtained.

An analogous relation for the local resistance is found by linearizing the gas-dynamic formula for the subsonic nozzle (attachment) with a smooth input, in which form all the other local resistances in the channel are represented. Using the formula

$$G_i = \mu F\sqrt{\frac{2\varkappa}{\varkappa-1}\frac{p_{i0}^2}{RT_{i0}}\left[\left(\frac{p_{i1}}{p_{i0}}\right)^{2/\varkappa} - \left(\frac{p_{i1}}{p_{i0}}\right)^{(\varkappa+1)/\varkappa}\right]},$$

a linearized dependence (in amplitudes of the dimensionless variations) is obtained for the local resistance

$$\delta\bar{G}_i = (1+\varepsilon_i)\delta\bar{p}_{i0} - \varepsilon_i\delta\bar{p}_{i1} - \frac{1}{2}\delta\bar{T}_{i0}, \quad (3)$$

where

$$\varepsilon_i = \frac{p_{i1}}{p_{i0}G_i}\frac{\partial G_i}{\partial(p_{i1}/p_{i0})}$$

is the slope of the dependence of the flow rate on the pressure ratio at the local resistance. If the local resistance described by the linearized relation in Eq. (3) is within (and not at the end of) the gas channel, it must also be described as a hexapole. To this end, it is assumed that the Joule–Thomson effect in the local resistance may be neglected, supposing that a constant temperature $T_{i0} = T_{i1}$ is maintained in the local resistance. Relating the temperature to the gas entropy by means of the thermodynamic equation $s = c_p \ln T - R\ln p +$ const and the other parameters by the continuity equation $\rho_{i0}u_{i0}F_{i0} = \rho_{i1}u_{i1}F_{i1}$, these relations are transformed to linearized form

$$\delta\bar{s}_{i0} + \frac{\varkappa-1}{\varkappa}\delta\bar{p}_{i0} = \delta\bar{s}_{i1} + \frac{\varkappa-1}{\varkappa}\delta\bar{p}_{i1}, \quad (4)$$

$$\delta\bar{\rho}_{i0} + \delta\bar{u}_{i0} = \delta\bar{\rho}_{i1} + \delta\bar{u}_{i1}. \quad (5)$$

The equation of the resistance as a hexapole is obtained from Eqs. (3)-(5) in matrix form

$$\begin{bmatrix} \delta\bar{u}_{i0} \\ \delta\bar{p}_{i0} \\ \delta\bar{s}_{i0} \end{bmatrix} = \begin{bmatrix} \frac{\varepsilon}{1+\varepsilon}\frac{u_0}{u_1} & \frac{\varkappa-1}{2\varkappa}\frac{1}{1+\varepsilon} & \frac{1}{2(1+\varepsilon)} \\ \frac{1}{1+\varepsilon}\frac{u_0}{u_1} & 1-\frac{\varkappa-1}{2\varkappa}\frac{1}{1+\varepsilon} & -\frac{1}{2(1+\varepsilon)} \\ -\frac{\varkappa-1}{\varkappa}\frac{1}{1+\varepsilon}\frac{u_0}{u_1} & \frac{(\varkappa-1)^2}{2\varkappa^2}\frac{1}{1+\varepsilon} & 1-\frac{\varkappa-1}{2\varkappa(1+\varepsilon)} \end{bmatrix} \begin{bmatrix} \delta\bar{u}_{i1} \\ \delta\bar{p}_{i1} \\ \delta\bar{s}_{i1} \end{bmatrix} = [A][\delta\bar{u}_{i1} \;\; \delta\bar{p}_{i1} \;\; \delta\bar{s}_{i1}]^T. \quad (6)$$

For the fork cross section in which gas is sampled from the channel, the extent (along the channel axis) of the outlet through which the gas is sampled is neglected, and it is assumed that the sections of channel before and after gas sampling are cylindrical in form, the mean gas velocity there is constant, and the Mach number of the flow is small. The latter assumption allows the pressure losses in the sampling cross section to be neglected and permits the assumption that the gas temperature and hence its entropy is maintained up to the sampling cross section, that is

$$\delta\bar{p}_{i1} = \delta\bar{p}_{(i+1)0}, \quad \delta\bar{s}_{i1} = \delta\bar{s}_{(i+1)0}. \quad (7)$$

The third equation for the sampling cross section, relating the gas flow rate before and after the sampling cross section, is written in the form of the continuity equation $G_i = G_{(i+1)} + G_{sa.i}$ or in linearized form

$$G_i\delta\bar{G}_i = G_{(i+1)}\,\delta\bar{G}_{(i+1)0} + G_{sa.i}\delta\bar{G}_{sa}. \quad (8)$$

Using Eqs. (4) and (5), the formulas for the amplitudes of the flow-rate variations are found, and then substituted into Eq. (8)

$$\begin{aligned} \delta\bar{G}_{i1} &= \delta\bar{u}_{i1} + (1/\varkappa)\,\delta\bar{p}_{i1} - \delta\bar{s}_{i1}, \\ \delta\bar{G}_{(i+1)0} &= \delta\bar{u}_{(i+1)0} + (1/\varkappa)\,\delta\bar{p}_{(i+1)0} - \delta\bar{s}_{(i+1)0}. \end{aligned} \quad (9)$$

The relation between the amplitude of flow-rate variation in sampling $\delta\bar{G}_{sa.i}$ with flow parameters in the sampling channel depends on the structural features of the channel. If the pressure losses in flow rotation may be neglected, a relation analogous to Eq. (9) may be written for this variation, and the resistance at the output from the sampling channel may be described as a hexapole using Eq. (6). If there is a local resistance at the input to the sampling channel, the flow-rate variation is found from Eq. (3) and its resistance is described by Eq. (6).

If there is not one but several sampling tubes leaving the fork, correspondingly a larger number of terms with flow-rate variations in the sampling tubes with their own coefficients will appear on the right-hand side of Eq. (8). If sampling is through a local resistance with a critical pressure difference (for example, a turbine nozzle), the variation in flow rate $\delta\bar{G}_{sa.i}$ is found from Eq. (3), which closes the system of hexapole equations. This case, being simpler, is considered here, and is the case for which the experiments described below are conducted. For this version, the fork in the sampling cross section is described by ten equations – three each for the two hexapoles before and after sampling, three for the parameter variations at the fork – Eqs. (7)-(9) – and Eq. (3) for the nozzle. In these ten equations, there appear 2 × 6 + 1 = 13 variables, since the same variables appear in Eqs. (7)-(9) as in the equations of the two hexapoles (except for $\delta G_{sa.i}$). Thus, the system of equations of the fork is unclosed.

To close this system of equations, the equations of the hexapoles describing the sections of the channel must be related to boundary conditions. Before proceeding to closure of the system of equations of the mathematical model of the channel, consider their simplification by bending the equations of the section of gas channel and the adjacent resistance. Suppose that there are two hexapoles, for example, a section of gas channel and a local resistance, one of which is at the output from the other. For these channel elements, dependences may be written in the form of hexapole Eqs. (2) and (6) and the conditions at their junction in the form of conditions of equal amplitude of the deviations of the flow parameters

$$\delta\bar{p}_{i1} = \delta\bar{p}_{(i+1)0}, \quad \delta\bar{u}_{i1} = \delta\bar{u}_{(i+1)0}, \quad \delta\bar{s}_{i1} = \delta\bar{s}_{(i+1)0}, \quad (10)$$

following from the assumption made that all the pressure losses are concentrated in local resistances, and the gas velocity is relatively small.

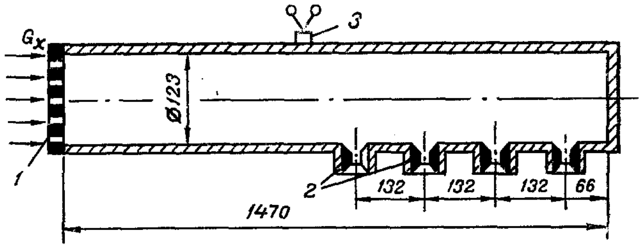

Fig. 1. Scheme of flow section of experimental apparatus: 1) grid for air; 2) output nozzles; 3) pressure center.

Using Eq. (10), Eqs. (2) and (6) may be reduced to a single equation, the matrix of which is equal to the product of matrices of the two initial hexapoles. In the same way, an equation for the channel from the input to the fork, between forks, and from the last fork to the output may be obtained from the conditions in Eq. (10) for all the junction cross sections between successive sections of the channel described as hexapoles. Here each of these channel sections is reduced to a hexapole equation. In the three equations of each hexapole, there are three undetermined variables. Their number is independent of the number of hexapoles and sampling points present in the system if, as assumed, all the sampling is through a nozzle with critical outflow. Therefore, three boundary conditions are required for closure of the system of equations.

The form of the equations relating the parameters at the channel input depends on the method of gas (working medium) supply or formation in the input cross section of the channel. It is assumed that nonisothermal gas flow at the input is formed (as a minimum from two flows) as a result of combustion or of chemical reaction or of the mixing of gases at different temperatures. The temperature of the gas formed at the input to the first section of the channel T_{10} depends on the relation between the gas flow rates G_0 and G_g: $T_{10} = f(G_0/G_g)$. The time of chemical reaction (combustion, mixing) and the volume of the reaction zone are neglected. The balance equation of the gas mass G_{10} formed from the flow rates G_0 and G_g, $G_{10} = G_0 + G_g$, taking account of the equation of state of the ideal gas and formulas relating the flow rate and the gas density and velocity in linearized form, is

$$\delta\bar{G}_{10} = \bar{G}_0\delta\bar{G}_0 + \bar{G}_g\delta\bar{G}_g = \delta\bar{u}_{10} + \delta\bar{p}_{10} - \delta\bar{T}_{10}. \tag{11}$$

Analogously, using Eq. (4) and linearizing the dependence for T_{10}, the amplitude of temperature variation δT_{10} is found as a function of the amplitudes of the flow-rate deviations $\delta\bar{G}_0$ and $\delta\bar{G}_g$ and the gas parameters

$$\delta\bar{T}_{10} = \psi(\delta\bar{G}_0 - \delta\bar{G}_r) = \delta\bar{s}_{10} + \frac{\varkappa - 1}{\varkappa}\delta\bar{p}_{10}. \tag{12}$$

Equation (12) is the first boundary condition at the input. Substituting Eq. (12) into Eq. (11), the second boundary equation is found

$$\delta\bar{p}_{10} + \delta\bar{u}_{10} = (\bar{G}_0 + \psi)\,\delta\bar{G}_0 + (\bar{G}_g - \psi)\,\delta\bar{G}_g. \tag{13}$$

The boundary conditions in Eqs. (12) and (13) include not only the parameters of the gas flow at the input but also the amplitudes of the external perturbations $\delta\bar{G}_0$ and $\delta\bar{G}_g$. The third and last boundary condition is also determined by the construction of the gas channel. If a nozzle (local resistance) is established at the output of the gas channel, Eq. (3) is used as the boundary condition; if there is a dead end at the end of the last n-th section of the channel, however, or the channel with sampling points takes the form of an annular (toroidal) collector, the condition of zero velocity $\delta\bar{u}_{n1} = 0$ holds at the end (midpoint of the ring) of the channel.

The resulting closed system of algebraic equations describing the gas flow in the whole of the given channel is solved on a computer by the Gauss method. It must be noted again that the given mathematical model is valid at relatively small Mach numbers ($M < 1$), when the pressure losses due to friction may be neglected, and until the one-dimensional approximation may be used. In addition, the model is applicable for the calculation of the dynamic characteristics of channels in the frequency range in which entropy-wave scattering on account of turbulent diffusion may be neglected [4].

To test the linear mathematical model developed for nonisothermal flow, experiments are conducted on an apparatus in which acoustic and entropy waves are excited simultaneously. A

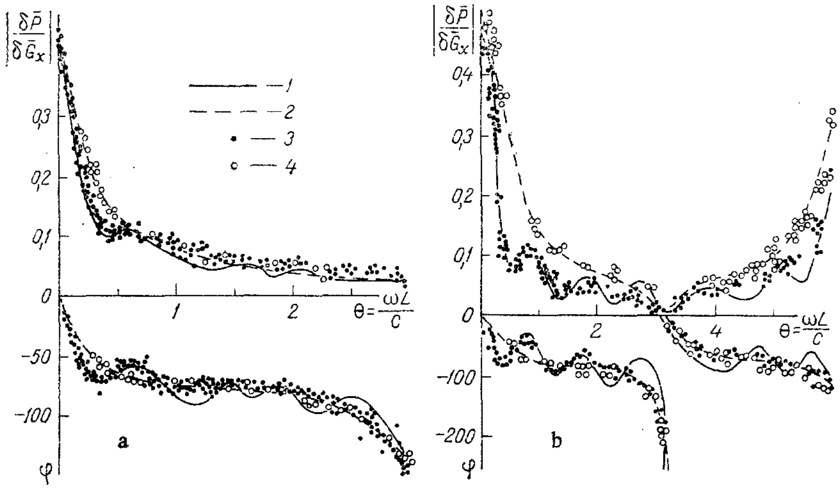

Fig. 2. Transfer amplitude and phase frequency characteristics of the flow in a channel with sampling points (a) and with a concentrated output at the end (b): 1) calculation, nonisothermal flow; 2) calculation, isothermal flow; 3) experiment, nonisothermal flow; 4) experiment, isothermal flow. φ, deg.

channel of cylindrical form ($L \sim 1.5$ m, $D = 0.12$ m; Fig. 1) has four short nozzles 2 at the side, uniformly spaced along the second half of its length. At the channel inlet, cold and hot air is fed independently through grid 1 with a large number of apertures (~200). The force pumps for the cold and hot gas are positioned alternately and uniformly over the channel cross section. A supercritical pressure difference is maintained there, so that oscillations in the channel have no influence on the flow rate through the force pumps. In the cold-air supply channel, a choke pulsator is established, creating harmonic pressure oscillations in front of the force pumps for cold-air supply. Since pressure variation in the given channel has no influence on the cold-air flow rate through the force pumps, the amplitude of the pressure variations in front of the force pumps is proportional to the amplitude of oscillations of the cold-air flow rate $\delta\overline{G}_{co}$. The presence of oscillations of the cold-air flow rate with simultaneous maintenance of a constant hot-air flow rate leads to the formation of nonsteady nonisothermal gas flow with entropy (temperature) waves.

The small permeability of the mixing grid at the channel input and the high gas velocity at the output cross sections of the grid jets ensure intensive mixing of the jets of cold and hot air close to the grid. Consequently, the mixing time and volume of the zone in which mixing occurs may be neglected. This is confirmed by the results of comparing the calculation results with the experimental data below. The mean gas velocity in the section from the channel input to the first output nozzle is ~65 m/sec, which corresponds to $M \simeq 0.15$.

In experiments using DDI-21 low-inertia inductive pressure sensors, the pressure variation of the cold air in front of the force pumps and the pressure oscillations of the gas in the channel at a distance $x = 0.69$ m from the grid (approximately at the midpoint of the channel) are measured. The readings of the pressure sensors and the rotational sensor of the pulsator are recorded on a magnetic recording unit; the experimental data are analyzed using a computational complex including a tracking filter and an M-6000 computer. In the course of the experiments, both the dynamic characteristics of nonisothermal flow (i.e., flow with entropy waves) and the purely acoustic characteristics of the channel for the case when cold air, some of it pulsating, is fed through both lattices are determined.

In addition, for comparison, experiments are performed with a channel of the same geometry (Fig. 1) but with no sampling points and with a concentrated output — only one nozzle at the end of the channel (in its end wall) with an area equal to that of the four nozzles in the distributed output. The experimental data are shown in the form of transfer amplitude and phase frequency characteristics $\delta\overline{p}(x_1)/\delta\overline{G}_{co}$ for the cross section with the coordinate $x_1 = 0.69$ m in Fig. 2a (dark points). Comparison of the experimental points with the results of calculations for nonisothermal flow shows that they are in complete agreement up to a dimensionless frequency $\Theta = \omega L/c \approx 1$. For higher frequencies, the dark points for nonisothermal flow practically coincide with the light points corresponding to isothermal flow. The explanation for this is partial scattering of the entropy waves [2, 4]. At relatively low frequencies in the experimental apparatus, it is explained by the above-noted overestimated level of turbulence, associated with the features of gas supply in the channel. The coefficient of diffusional turbulence in the channel according to the measurements is higher by an order of magnitude than its usual value for a turbulent flow. On the curves of the amplitude and phase characteristics of the nonisothermal flow, the typical feature of curves of the frequency characteristics of channels with entropy waves is seen: the presence of "waves" associated with the interaction of entropy and acoustic effects at the output resistance. The good agreement of the experimental data with the calculation results confirms the efficiency of the mathematical model developed for the channel with gas sampling points.

In Fig. 2b, the frequency characteristics of a channel with a concentrated output (without sampling points over the length of the channel), with a nozzle at the end of the channel, are shown. Experiments and calculations are undertaken to elucidate the influence of sampling points does not lead to qualitative change in the character of the dynamic characteristics of the channel. It is noteworthy that, in the case of a concentrated output (Fig. 2b), the amplitude of the "waves" in the amplitude and phase frequency characteristics is larger than for a channel with samplings points (Fig. 2a). This is explained in that, with a distributed output, the interaction of acoustic and entropy waves is reduced, since it is concentrated in different sampling cross sections, at which the entropy waves arrive with different phase.

NOTATION

D, L, diameter and length of channel; u, mean gas velocity; p, ρ, mean gas pressure and density; c, sound velocity in gas; M, Mach number; δp, δu, dimensionless variation (deviation) in pressure and density, referred to mean values of variable parameters; δs, dimensionless variation in gas entropy, referred to specific heat of gas at constant pressure; $\delta\bar{p}$, $\delta\bar{u}$, $\delta\bar{s}$, amplitude of variation in flow parameters; ω, angular frequency of induced oscillations; $\delta\bar{u}_{i0}$, $\delta\bar{p}_{i0}$, $\delta\bar{s}_{i0}$, amplitude of dimensionless flow parameters at the input to the i-th section; $\delta\bar{u}_{i1}$, $\delta\bar{p}_{i1}$, $\delta\bar{s}_{i1}$, the same for output from the i-th section of the channel; T_{i0}, T_{i1}, temperature at the input and output of the section of the channel (local resistance); κ, adiabatic index of gas; u_0, u_1, gas velocity at input and output of section with local resistance; G_i, $G_{(i+1)}$, mean gas flow rate in i-th and (i + 1)-th sections; $G_{sa.i}$, mean gas flow rate in sampling from i-th section; μ, flow-rate coefficient; $\delta\bar{G}_{i1}$, $\delta\bar{G}_{(i+1)0}$, $\delta\bar{G}_{sa.i}$, amplitude of variation in these quantities; G_0, G_g, flow rates of the media from which the gas is formed at the input; $\delta\bar{G}_0$, $\delta\bar{G}_g$, amplitudes of their dimensionless variation; $\delta\bar{T}_{10}$, $\delta\bar{u}_{10}$, $\delta\bar{p}_{10}$, $\delta\bar{s}_{10}$, amplitudes of the parameter variation at the input to the first section of the channel; $\bar{G}_0 = G_0/(G_0 + G_g)$; $\bar{G}_g = G_g(G_0 + G_g)$; $\psi = G_0(G_gT)[\partial T/\partial(G_0/G_g)]$; $\delta\bar{p}(x)/\delta\bar{G}_{co}$, transfer frequency function of the channel for pressure oscillations in the cross section with coordinate x_1 in the case of perturbation by change in the cold-air flow rate $\delta\bar{G}_{co}$.

LITERATURE CITED

1. B. F. Glikman, Nonsteady Flow in Pneumohydraulic Circuits [in Russian], Moscow (1979).
2. B. F. Glikman, Mathematical Models of Pneumohydraulic Systems [in Russian], Moscow (1986).
3. L. Zade and Ch. Dezoer, Theory of Linear Systems [Russian translation], Moscow (1970).
4. B. F. Glikman and V. A. Gur'ev, Izv. Akad. Nauk SSSR, Energ. Transp., No. 1, 149-155 (1979).